

# Abstract

This thesis deals with different design schemes of variable structure control (VSC) based flight control systems to the aerospace vehicles, primarily for missiles. The major contributions of the thesis are towards the development of new theories in VSC for discrete-time systems using state feedback and output feedback. Most of the flight/launch vehicle models are nonlinear and time varying in nature. The aircraft or missile flight control system must provide stability, disturbance rejection, and reference signal tracking, while their aerodynamic coefficients vary over a wide range due to large mach-altitude variations, non-linear characteristics of missile or aircraft dynamics and uncertainties resulting from inaccurate vehicle data. The flight vehicle model refined using the wind tunnel generated data may not still be accurate enough. Model inaccuracies also result from idealization and linearization of equations governing the physical system. The controller must ensure stability and robustness irrespective of the model uncertainties, parameter variations during flight and external disturbances. Since the variable structure control is well-known for its robustness properties, it is used for the flight controller design. The recent developments in variable structure systems theory have produced highly attractive synthesis approaches in both continuous and discrete-time domains. The controller structure naturally takes into account of the external disturbances and uncertainty inherent in the plant model.

Variable structure control is a robust nonlinear control strategy characterized by a discontinuous feedback control structure that is switched as the system state crosses certain user specified surfaces called switching surfaces (SS) denoted by  $\sigma_k$ . The control law is selected to steer the system state/output trajectory from any initial point onto the SS and there after to constrain the trajectory motion on the SS or in the close vicinity of the SS. When the system trajectory is moving on the SS, (i) the system is said to be in sliding mode (SM) and it behaves as a reduced order system (ii) the system is invariant to parameter variations and external disturbances which are matched. The SS is designed

so that the system will have desired characteristics on the SS. Thus the design of a proper switching surface is the key issue in variable structure control systems (VSCS). The most attractive feature of VSC is the SM, during which the closed loop system behaves like a reduced order dynamics (ROD). The ROD can be obtained from the condition  $\sigma_k = 0$ , together with the system equations. The VSC based flight controller works in two phases: reaching phase and sliding phase. The reaching phase is designed such that the system state from any initial condition (corresponding to a reference input for tracking systems) is driven onto the switching manifold in finite time. Once the system trajectory comes to the switching surface, it is constrained to slide along the SS. The general form of the VSC law is  $u = u_{eq} + u_d$  where  $u_{eq}$  is the equivalent control input i.e., the linear part of total control and  $u_d$  is nonlinear/discontinuous part of the control law. This thesis highlights on the following topics:

1. Tracking controller design in continuous and discrete time domain, with state feedback as well as with output feedback (OFB)
  - The optimal design of switching surfaces (particularly, SS of the form  $\sigma_k = Sx_k$  and  $\sigma_k = Sx_k - S_r r_{k-1}$ )
  - Reaching phase and sliding phase design
2. Discrete VSC (DVSC) with sliding sector design using dissipativeness property of the system for both state feedback and output feedback
3. Discrete variable structure (DVS) controller design for system with matched and mismatched uncertainties
4. DVSC with direct (static) OFB and with dynamic compensator
5. Simulation studies of the nominal system (with CVSC and DVSC), and also of system in the presence of external disturbances and wind gust input
6. Chattering, accuracy and robustness issues

Literature survey includes the various design aspects of VSC, reaching and sliding modes, DVSC using state and output feedback, and applications of VSC, along with a good number of references. Chapter 2 presents the open loop responses of a dual input air-to-air missile at four operating points corresponding to mach numbers  $M = 1.2, 1.5, 2$  &  $4$ . These responses indicate that the missile is open loop stable but has non-minimum phase characteristics and the short period damping is very poor. A robust variable structure tracking controller with full state feedback in continuous-time domain, is designed in chapter 3, for the missile pitch autopilot, in order to demonstrate the feasibility of VSC to the flight control applications. The optimal SS is designed by LQR method so that the

ROD will have the desired characteristics as stability and tracking. The simulation study shows the feasibility of using VSC for missile autopilot design. The variable structure control techniques being robust control method, offer bright promise for practical flight control systems design as well as other industrial applications.

A discrete version of VSC at finite sampling time enhances chattering with the pre-designed SM and even may cause instability with large gains, and need modification of SS as well as control law designs. A missile pitch autopilot is designed in chapter 4, using DVS controller with two types of switching surfaces, (i) a linear state dependent SS  $\sigma_k = Sx_k$ , and (ii) a linear reference command depend SS  $\sigma_k = Sx_k - S_r r_{k-1}$ , assuming all states are available for feedback. The matrix  $S$ , which define the SS, is obtained by applying linear quadratic regulator (LQR) theory to the ROD, when the system is sliding on the SS. The tracking problem is taken into consideration while computing  $S$ , by adding an extra cost term in terms of tracking error in the performance index, so as to improve the transient response of the system. It is clear from the simulation responses of the nominal system that the state trajectories remain on the switching surface  $\sigma_k = 0$ , whereas for system in the presence of uncertainties, the state trajectories deviates from the SS,  $\sigma_k = 0$ , but remain in the close vicinity of SS. This requires a nonzero  $u_d$ , throughout the control period.

In DVSCS, a 'switching region' or a 'sliding lattice' exists in the neighborhood of the sliding surface, instead of the conventional switching surface in CVSCS. To take into account of this fact, a DVS controller is designed with a sliding sector. The sliding sector is a region around the reference command depend SS. A new algorithm is developed for the variable width sliding sector design, by considering the dissipativeness property of the system. The advantage of using this approach is that it deals with the input-output energy, hence it is an appropriate concept for a tracking problem. The sliding sector is designed such that the system is dissipative inside the sector. A linear control law is used inside the sector. This sliding sector can be considered as a variable width sector whose width depends on the norm of the system state and the reference input. The sliding sector around the SS helps in completely eliminating the chatter and also to use a zero discontinuous control component. The DVS controller is designed with sliding sector for uncertain systems with additive, matched as well as mismatched uncertainties. The sufficient conditions on the norm-bound of the uncertainty are given for robust stability in the sense of Lyapunov. These conditions ensure reaching and sliding. The DVS control law drives the state trajectories into the sliding sector and is constrained to stay inside it thereafter, even in the presence of bounded matched and mismatched uncertainties. The main advantage with this DVS controller is that for the implementation, the upper bound on the matched and mismatched uncertainties need not be known beforehand, instead only the norm-bound conditions required to be satisfied. In order to show that

VSC is applicable to challenging problems like non-piloted, remotely controlled air vehicles, a DVS controller is designed for the stabilization of a MAV, which is developed in the Department of Aerospace Engineering, Indian Institute of Science, Bangalore. These tiny non-piloted air vehicles are susceptible to external disturbances and often have poor stability and handling/flying qualities. The control objective is to design a controller to achieve automatic stabilization and to improve short period damping of the MAV.

Output feedback in VSCS is desirable because it requires fewer feedback loops and is much cheaper and simple to implement. The existing variable structure output feedback control (VSOFC) design algorithms require invertibility of certain matrix product  $SC\Gamma$ , which in turn require  $CB$  to be full rank, for the implementation of the control law, which are not met in the case of a irregular plant. In this thesis, for the missile model, the plant input matrix  $\Gamma$  and plant output matrix  $C$  are such that  $C\Gamma = 0$  or  $\text{rank}(C\Gamma) < m$  and hence  $SC\Gamma = 0$ . The traditional equivalent control law does not exist in this case. By including additional outputs which are the delayed states of a computational delay unit (generated inside the controller),  $C\Gamma$  is made to have full rank. Two types of controllers, based on discrete VSOFC (DVSOFC), are designed in this thesis (chapter 5) using static OFB (SOFB) and with dynamic compensator. In the former case, the optimal switching surface and in the latter, the compensator parameters are determined by LQR output feedback technique.

A missile pitch autopilot is designed based on DVSOFC (with a SS  $\sigma_k = Sy_k$ ), using SOFB. The VSC with SOFB has certain limitations, like the use of right inverse of the output matrix,  $C^{-R}$  in the equivalent control law  $u_{eq}$ , will not ensure the assigned eigenvalues to the ROD and this  $u_{eq}$  may not give an ideal sliding motion on the SS. Hence a sliding sector around the SS is designed in terms of the system outputs and reference command. A VSC law is used to drive the trajectories towards the sector and a linear control law is used inside the sector. The inherent problems with output feedback in linear systems are also present in VSOFC, like, it may not be possible to stabilize a system and also to get good performance with a given number of outputs using SOFB. In that case one should opt for VSC with a dynamic compensator. In general, the need to use extra dynamics results from the relative dimensions and structural properties of the system. Missile pitch autopilot using DVSOFC with a dynamic compensator having a particular structure is also designed. The sliding function is defined in terms of the compensator states, hence the invertibility of  $(SC\Gamma)$  is not required. Both the controllers (with SOFB and dynamic compensator) shows perfect tracking capability even in the presence of uncertainties and disturbances.

A dual-input (wing & tail), air-to-air, missile pitch autopilot design for tracking a

given lateral acceleration command is considered throughout the thesis using the various controllers described in each chapter. The controllers are simulated over a small section of the flight envelope consisting of operating points corresponding to mach numbers 1.2, 1.5, 2 and 4. It is observed from the simulation study (nominal system) of various controllers that in the case of full state feedback, the trajectories remain on the switching surface  $\sigma_k = 0$ , whereas in the OFB case the trajectories move towards the SS to remain in the close vicinity of  $\sigma_k = 0$ . From the simulation responses of the controller with state feedback, it is seen that the controller designed at one particular operating point works well for all other operating points. The simulation results show excellent performance and capabilities of the control system structure. The simulation study with matched and mismatched uncertainties, noise and wind gust input indicates that the controllers designed are robust with respect to parameter variations and external disturbances. In all the cases, the controller is able to track a given lateral acceleration as high as 20g without any steady state tracking error. There is no undershoot in the  $a_z$  response as expected in the case of non-minimum phase systems, since major component of lateral acceleration is produced by the wing.

Comparing the responses of system (nominal case) at  $M=1.5$ , with state feedback controllers with SS (i)  $\sigma_k = Sx_k$ , (ii)  $\sigma_k = Sx_k - S_r r_{k-1}$ , and with sliding sector, it is observed that all the controllers are able to track a step command of 20g without any undershoot. Among the three, the controller with sliding sector gives better responses, with a least settling time of 0.168sec and a small rise time of 0.075sec. Also with VSOFB, the controllers give comparable results. The system responses are equally good as in the case of state feedback. At  $M=1.5$ , the system responses with controller (VSOFB) using sliding sector are corresponding to a settling time of 0.198sec and a rise time of 0.115sec. In the case of dynamic compensator, the settling time is 0.1854sec and rise time is 0.1sec. Thus, the VSOFB controller can replace the state feedback controller at the cost of small increase in settling time but the implementation will be easy in output feedback due to limited number of measurements. In the case of OFB, the controller design needs tuning of the parameters for each system model at different operating point.

This thesis aims to make contributions towards VSC theory and its applications to the flight controller design. The different schemes evolved in the thesis for the design of variable structure controllers in state/output variable space, using SS/sliding sector concept helps in attaining high performance, improving robustness and eliminating chattering. From the over all study carried out in this thesis, it can be emphasized that VSC gives robust performance and is best suited for the flight controller design of missile/aircraft.

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